

# Policy Gradient

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Reinforcement Learning Summer School

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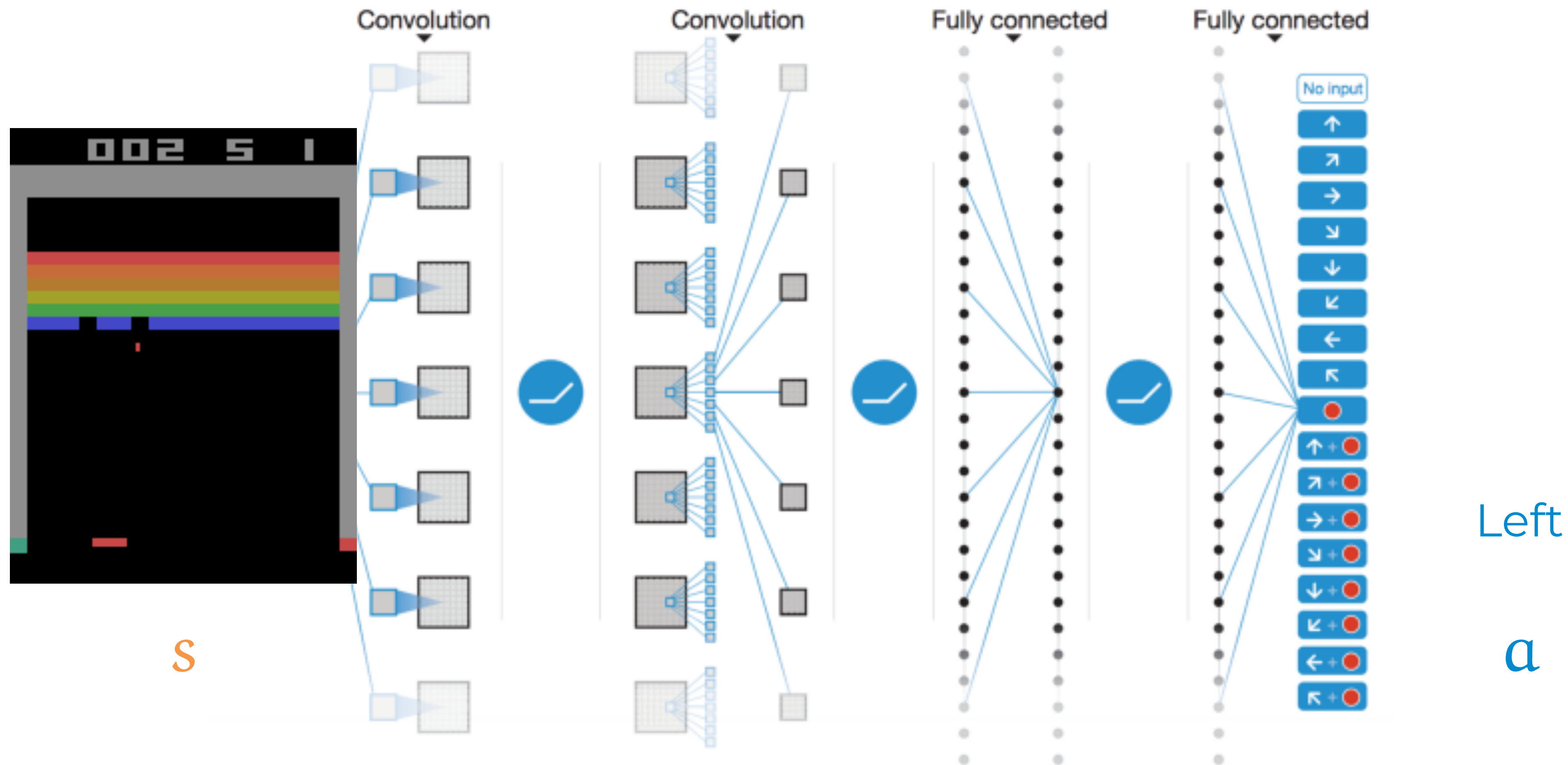
## Deep Reinforcement Learning setting

- Neural network **policies**
- Model-free
- On-policy

## Overview

- Deriving REINFORCE
- Actor-critic
- Advanced methods
  - TRPO, PPO
  - Soft Actor-Critic
  - World models

# ATARI POLICY



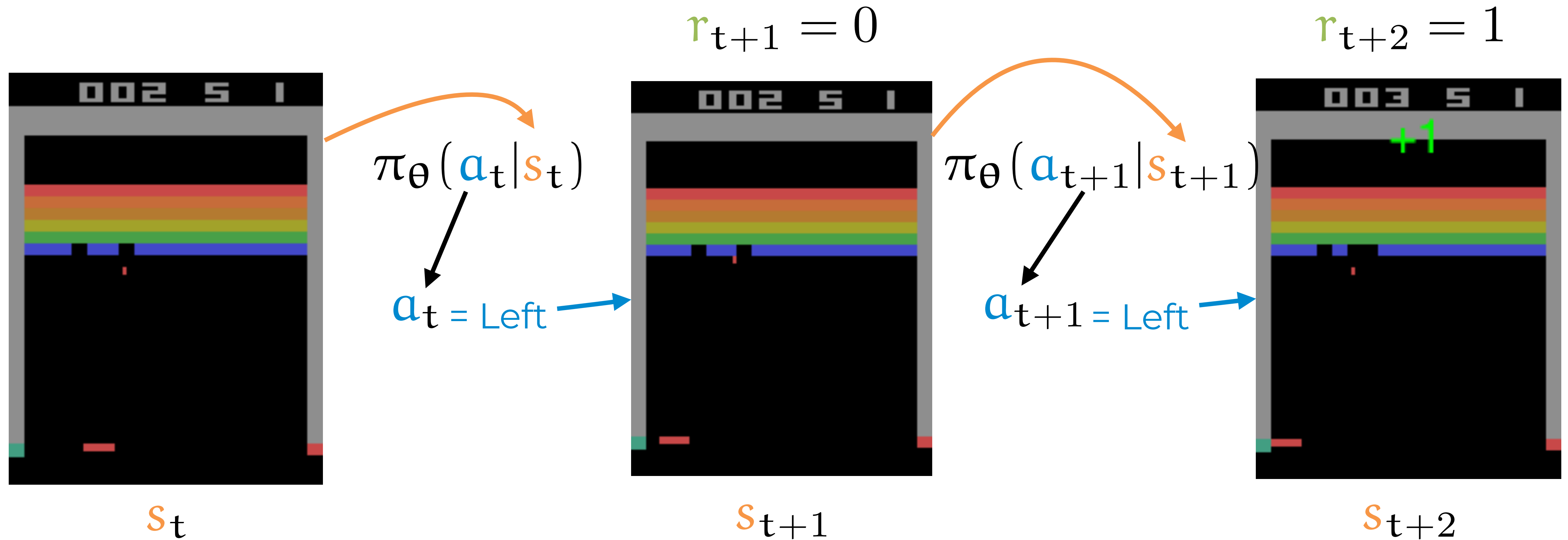
CNN policy network  $\pi_{\theta}(a|s)$

Mnih, V., Kavukcuoglu, K., Silver, D. et al. Human-level control through deep reinforcement learning. *Nature* 518, 529–533 (2015).



- Components of RL:
  - Actions  $a_t$
  - States  $s_t$
  - Rewards  $r_t$
- These are random variables!
- Trajectories  $\tau = s_0, a_0, r_1, s_1, a_1, r_2, s_2 \dots a_{T-1}, r_T, s_T$
- Initial state  $s_0$
- Terminal state  $s_T$

# ATARI TRAJECTORY



**Credit assignment:**  
What **action** causes **reward**?

<https://becominghuman.ai/lets-build-an-atari-ai-part-0-intro-to-rl-9b2c537640ec>

Trajectories  $\tau = s_0, a_0, r_1, s_1, a_1, r_2, s_2 \dots a_{T-1}, r_T, s_T$

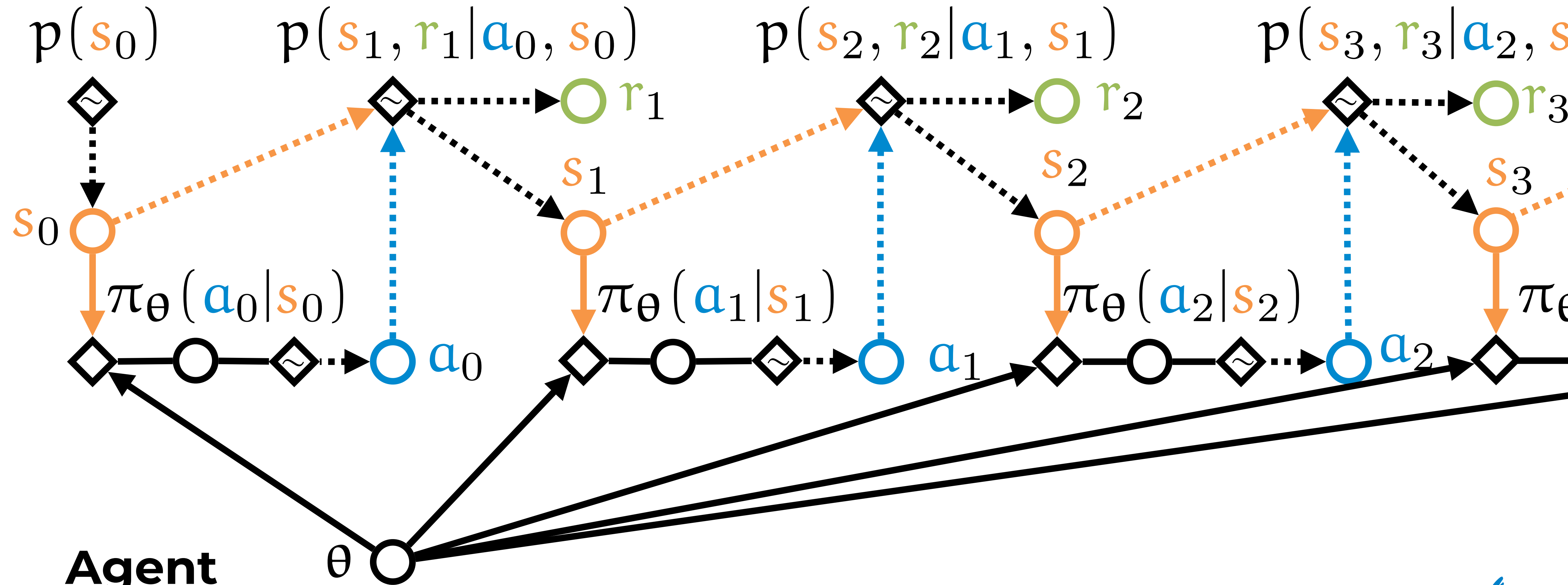
Markov decision processes (MDPs):

$$p(\tau|\theta) = p(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t) p(s_{t+1}, r_{t+1}|s_t, a_t)$$

- **Policy**  $\pi_{\theta}(a_t|s_t)$
- **State** transition distribution  $p(s_{t+1}, r_{t+1}|s_t, a_t)$
- Initial **state** distribution  $p(s_0)$

# MARKOV DECISION PROCESSES

## Environment



Agent

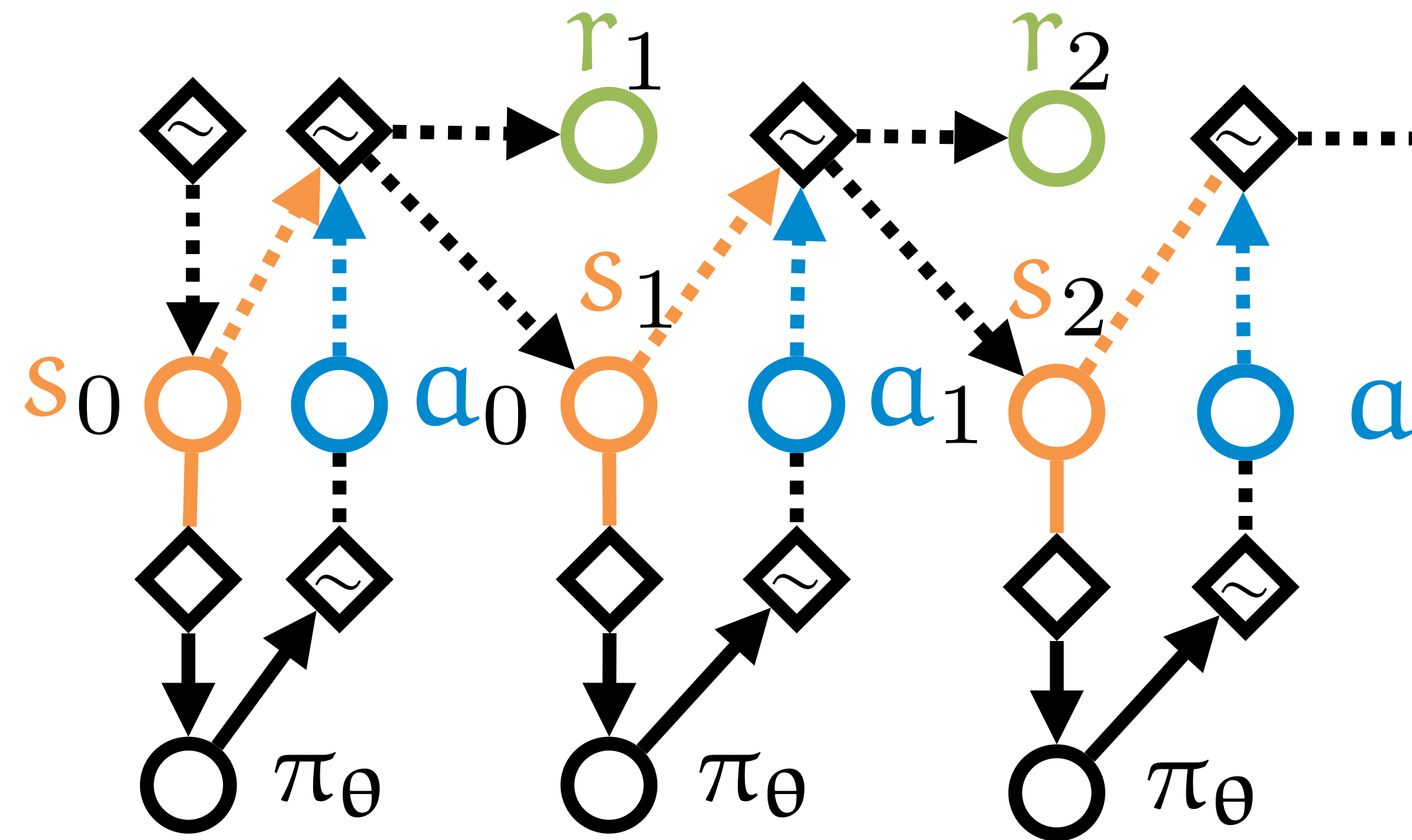


# OBSERVABILITY

- Full observability of **state**



- Partial observability: POMDP
- Out of scope!

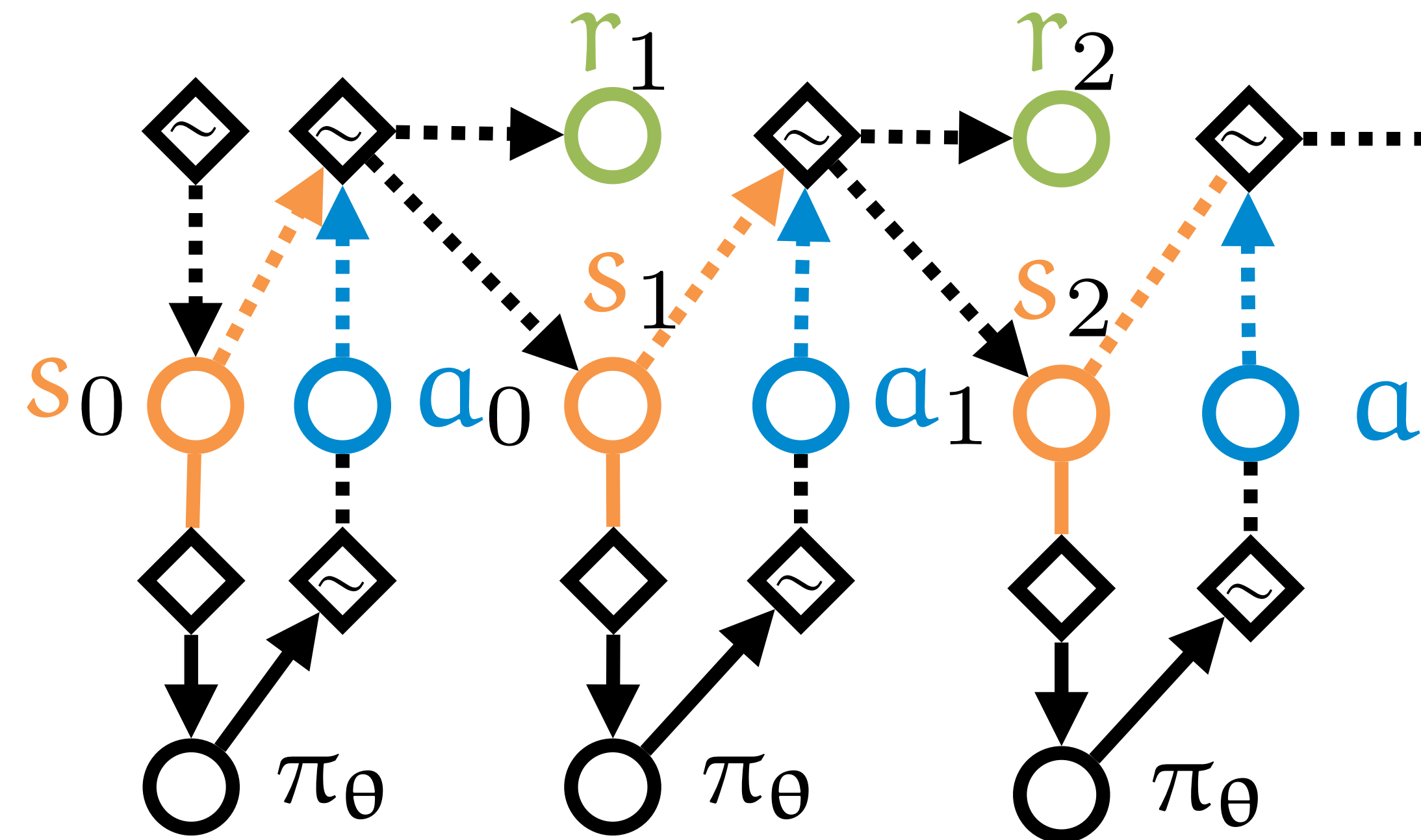


# MARKOV ASSUMPTION

$s_t$  independent of history  
given  $s_{t-1}$ :

$$p(s_t | a_{t-1}, s_1, \dots, s_{t-1}) = p(s_t | a_{t-1}, s_{t-1})$$

- Used to derive strong algorithms!
- Fundamental assumption behind RL
- No RL is completely “*model-free*”!



# EXPECTED RETURN

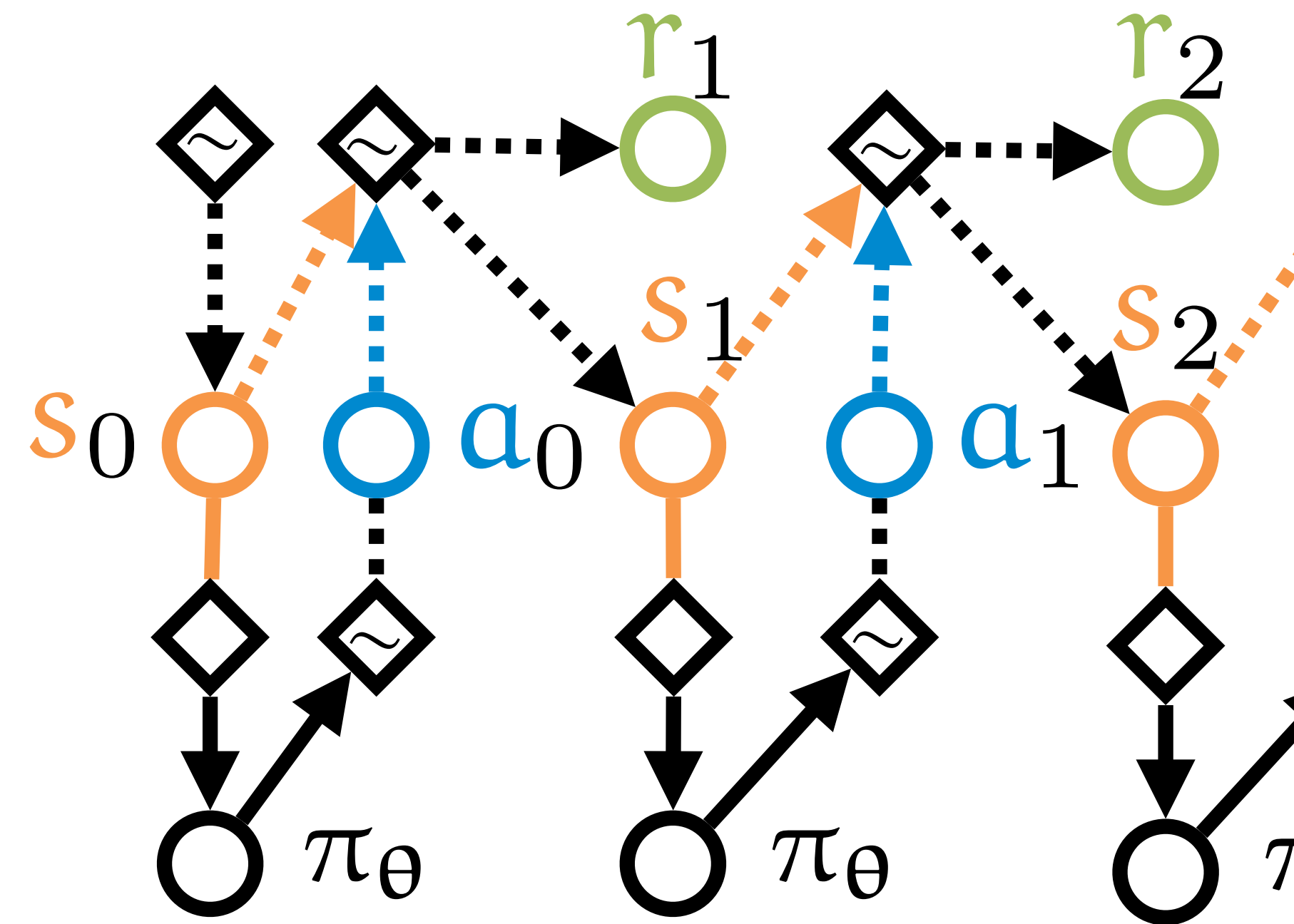
Total **discounted reward**  $0 \leq \gamma \leq 1$

$$R_\gamma = r_1 + \gamma r_2 + \gamma^2 r_3 + \dots + \gamma^{T-1} r_T$$

- **Rewards** and **states** are **stochastic!**
- **Goal:** Maximize **expected return**

$$J(\theta) = \mathbb{E}_{p(\tau|\theta)} [R_\gamma]$$

$$\theta^* = \arg \max_{\theta} J(\theta)$$





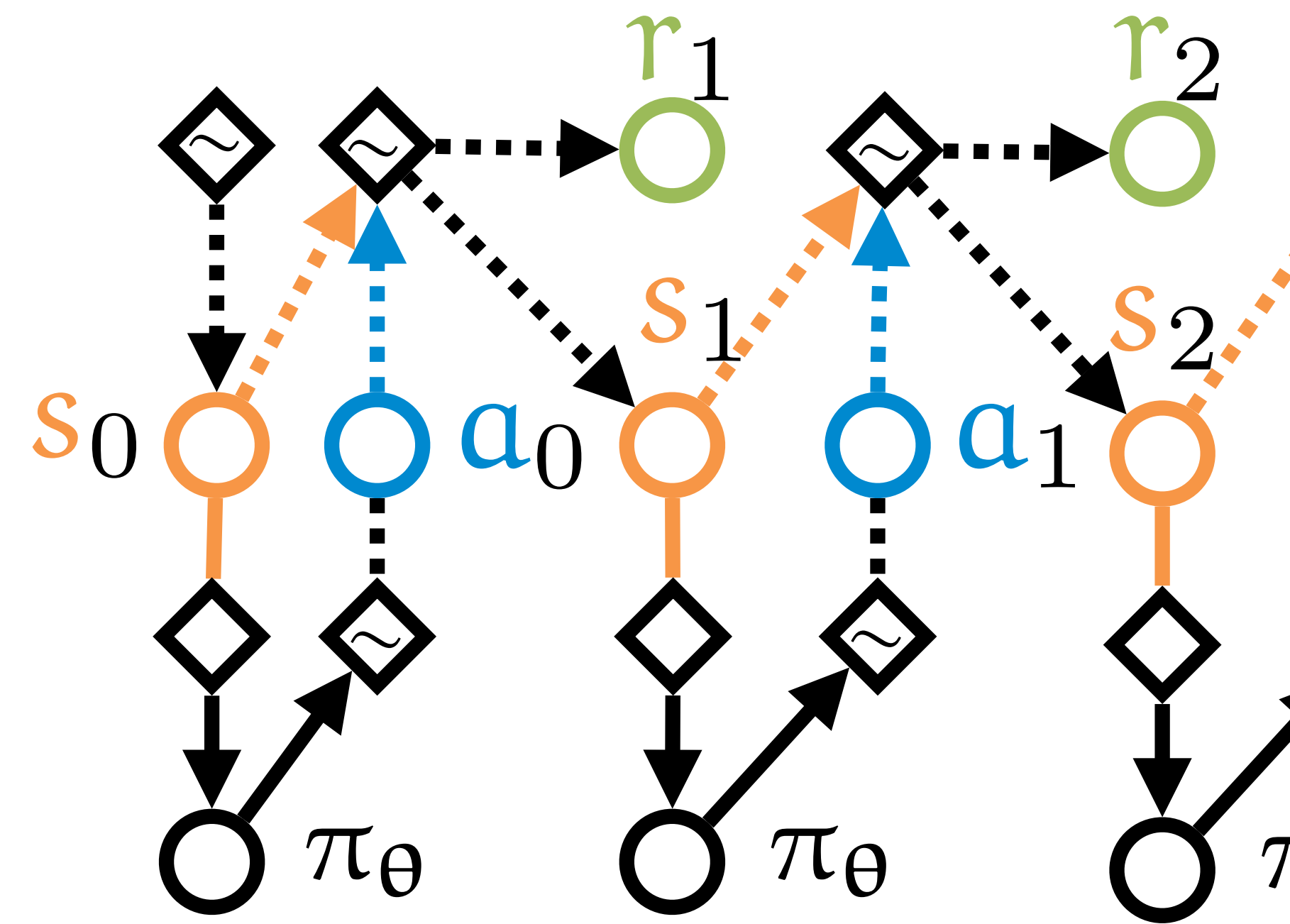
# EXPECTED RETURN

$$J(\theta) = \mathbb{E}_{p(\tau|\theta)} [R_\gamma]$$

Expectation over **trajectories** by following **policy**  $\pi_\theta$

Requires summing (or integrating) over *all* trajectories!

→ Monte Carlo (sampling) estimation





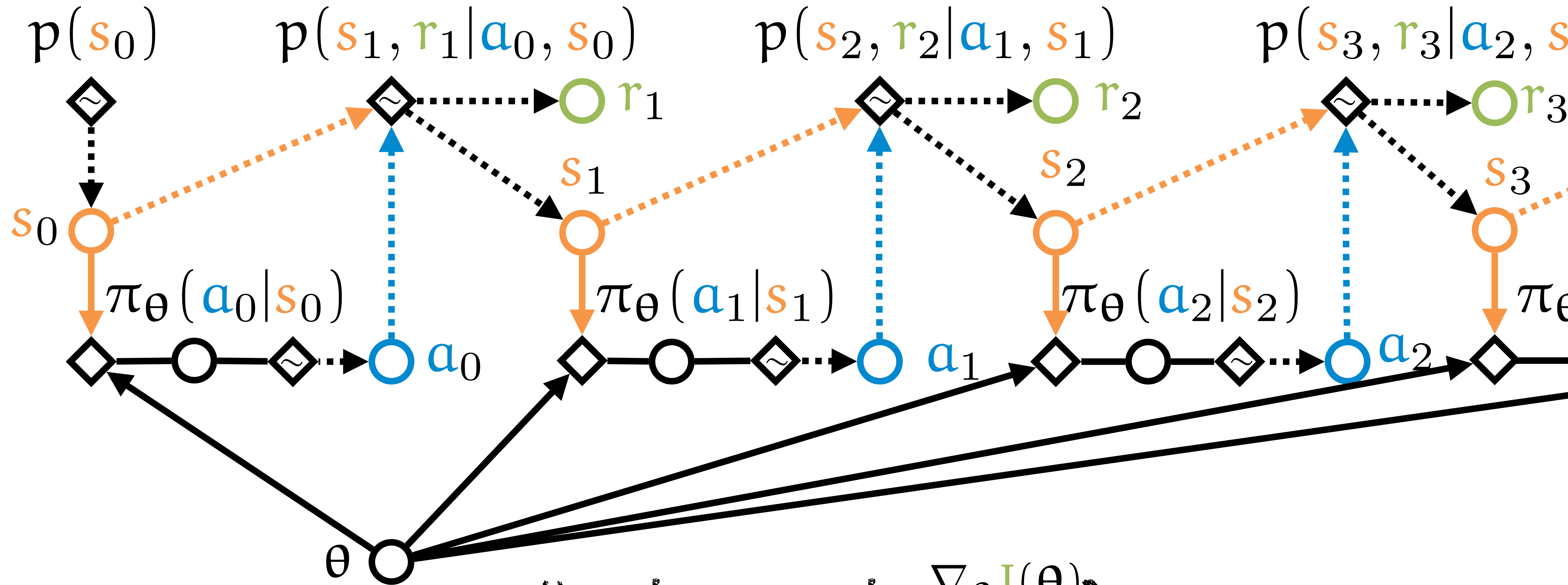
# MAXIMIZING EXPECTED RETURN

$$J(\boldsymbol{\theta}) = \mathbb{E}_{p(\boldsymbol{\tau}|\boldsymbol{\theta})}[\mathbf{R}_\gamma]$$

How to find  $\boldsymbol{\theta}^* = \arg \max_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$ ?

→ **Policy gradient methods:** Use  $\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$  in gradient *ascent*  
 $\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i + \alpha \nabla_{\boldsymbol{\theta}_i} J(\boldsymbol{\theta}_i)$

# MARKOV DECISION PROCESSES



How to compute  $\nabla_\theta J(\theta)$ ?  
Gradient estimation!

Simple REINFORCE:

1.  $\tau \sim p(\tau|\theta)$
2.  $\theta \leftarrow \theta + \alpha R_\gamma \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$

Sample trajectory

Gradient ascent

Let's derive algorithm!

# JOINT DISTRIBUTION OF MDP

$$\nabla_{\theta} J(\theta) = \nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}]$$

Expectation is over *all* trajectories  $\tau$  :

$$\nabla_{\theta} J(\theta) = \sum_{\tau} R_{\gamma} \nabla_{\theta} p(\tau|\theta)$$

To sample, we need an expression like

$$\sum_{\tau} p(\tau|\theta) f(\tau)$$

Solution: The **score function!**

# THE SCORE FUNCTION

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \sum_{\tau} R_{\gamma} \nabla_{\theta} p(\tau|\theta) \\ &= \sum_{\tau} R_{\gamma} \nabla_{\theta} p(\tau|\theta) \frac{p(\tau|\theta)}{p(\tau|\theta)} \\ &= \sum_{\tau} R_{\gamma} p(\tau|\theta) \frac{\nabla_{\theta} p(\tau|\theta)}{p(\tau|\theta)}\end{aligned}$$

Multiply by 1

This is an expression like  $\sum_{\tau} p(\tau|\theta) f(\tau)$  !

# THE SCORE FUNCTION

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \sum_{\tau} R_{\gamma} \nabla_{\theta} p(\tau|\theta) \\ &= \sum_{\tau} R_{\gamma} \nabla_{\theta} p(\tau|\theta) \frac{p(\tau|\theta)}{p(\tau|\theta)} \\ &= \sum_{\tau} R_{\gamma} p(\tau|\theta) \frac{\nabla_{\theta} p(\tau|\theta)}{p(\tau|\theta)} \\ &= \sum_{\tau} p(\tau|\theta) R_{\gamma} \nabla_{\theta} \log p(\tau|\theta) \\ &= \mathbb{E}_{p(\tau|\theta)} [R_{\gamma} \nabla_{\theta} \log p(\tau|\theta)]\end{aligned}$$

**Score function:**

$$\begin{aligned}\nabla_{\theta} \log p(\tau|\theta) &= \frac{\partial \log p(\tau|\theta)}{\partial p(\tau|\theta)} \frac{\partial p(\tau|\theta)}{\partial \theta} \\ &= \frac{1}{p(\tau|\theta)} \nabla_{\theta} p(\tau|\theta)\end{aligned}$$

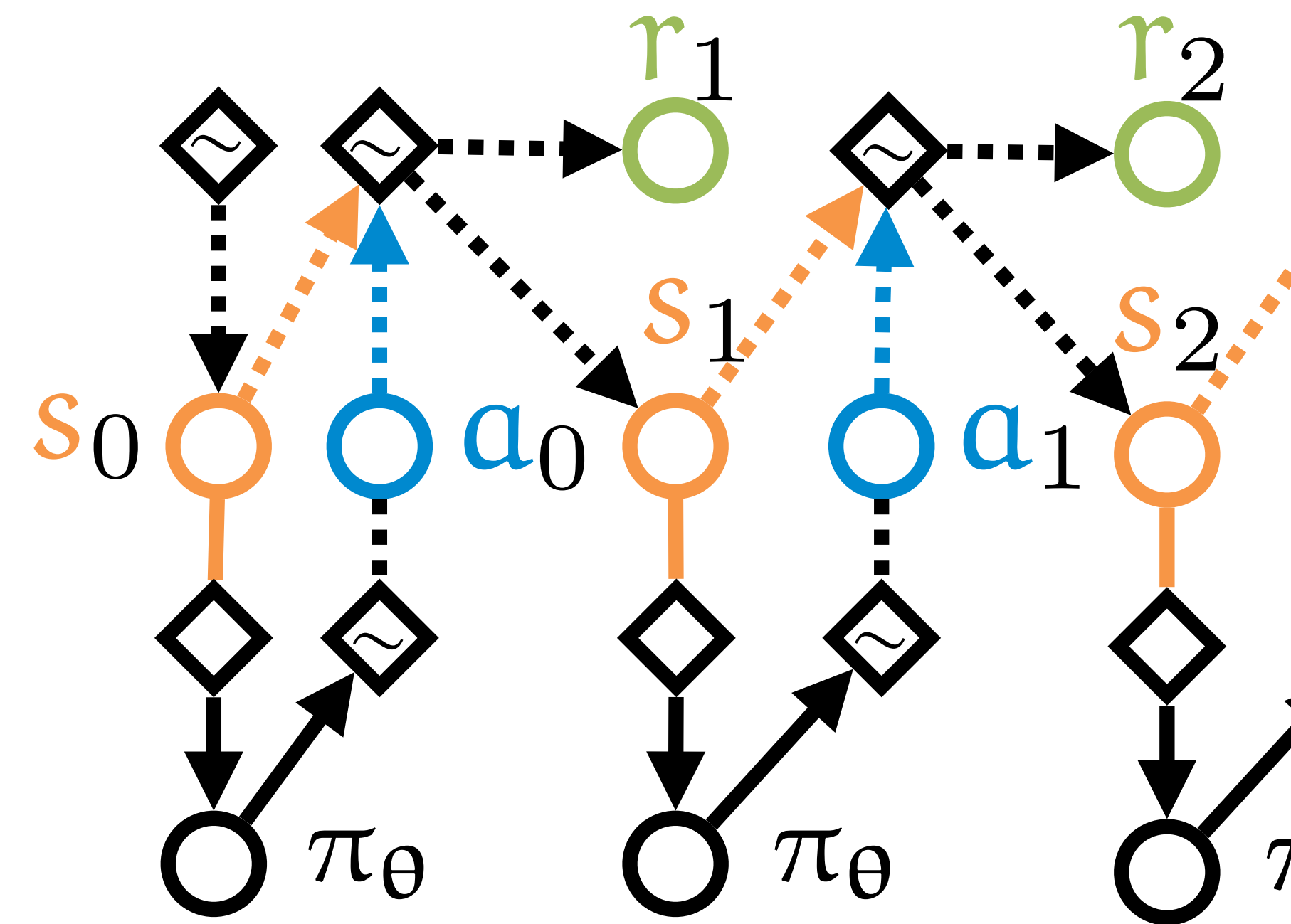
# WORKING OUT MDP

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{p(\tau|\theta)} [\mathbf{R}_{\gamma} \nabla_{\theta} \log p(\tau|\theta)]$$

How to compute  $\nabla_{\theta} \log p(\tau|\theta)$ ?

MDP distribution over trajectories:

$$p(\tau|\theta) = p(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t) p(s_{t+1}, r_{t+1}|s_t, a_t)$$



# WORKING OUT MDP

$$p(\boldsymbol{\tau}|\boldsymbol{\theta}) = p(\mathbf{s}_0) \prod_{t=0}^{T-1} \pi_{\boldsymbol{\theta}}(\mathbf{a}_t|\mathbf{s}_t) p(\mathbf{s}_{t+1}, \mathbf{r}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\log p(\boldsymbol{\tau}|\boldsymbol{\theta}) = \log p(\mathbf{s}_0) + \sum_{t=0}^{T-1} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t|\mathbf{s}_t) + \log p(\mathbf{s}_{t+1}, \mathbf{r}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\tau}|\boldsymbol{\theta}) = \nabla_{\boldsymbol{\theta}} \log p(\mathbf{s}_0) + \sum_{t=0}^{T-1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t|\mathbf{s}_t)$$

$$+ \nabla_{\boldsymbol{\theta}} \log p(\mathbf{s}_{t+1}, \mathbf{r}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$$

$$\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\tau}|\boldsymbol{\theta}) = \sum_{t=0}^{T-1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t|\mathbf{s}_t)$$

Gradient of environment wrt policy parameters  $\boldsymbol{\theta}$  is 0!



$$\nabla_{\theta} \log p(\tau|\theta) = \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{p(\tau|\theta)} [R_{\gamma} \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)]$$

$$\approx R_{\gamma} \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t), \quad \tau \sim p(\tau|\theta) \quad \text{Monte Carlo (sample) estimate}$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{p(\tau|\theta)} \left[ R_{\gamma} \sum_{t=0}^{\tau-1} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right]$$

**Reinforce** actions with high *total return*

- Reinforce  $\mathbf{a}_{\tau-1}$  when  $r_1$  is high?
- Only reinforce **actions** with good *consequences*!

Gradient of **reward** at  $t' + 1$ :

$$\begin{aligned} \nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [\gamma^{t'} r_{t'+1}] &= \mathbb{E}_{p(\tau|\theta)} [\gamma^{t'} r_{t'+1} \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta} (a_t | s_t)] \\ &= \mathbb{E}_{p(\tau|\theta)} [\gamma^{t'} r_{t'+1} \sum_{t=0}^{t'} \nabla_{\theta} \log \pi_{\theta} (a_t | s_t)] \end{aligned}$$

Only influenced by **actions** until  $t'$

Sum over timesteps:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [\mathbf{R}_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t'=0}^{T-1} \gamma^{t'} r_{t'+1} \sum_{t=0}^{t'} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right]$$

Equivalent: Update actions based on following rewards

- Discounted reward to go

$$\mathbf{G}_t = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'+1}$$

Gradient estimate:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [\mathbf{R}_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t \mathbf{G}_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right]$$

$$G_t = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'+1}$$

## REINFORCE:

1.  $\tau \sim p(\tau|\theta)$
2.  $\theta \leftarrow \theta + \alpha \sum_{t=0}^{T-1} \gamma^t G_t \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$

Sample trajectory

Gradient ascent

# ACTOR-CRITIC

## Variance:

$$\mathbb{V}[\mathbf{g}] = \mathbb{E}_{p_{\theta}} \left[ \sum_{i=1}^D (\mathbf{g}_i - \mathbb{E}_{p_{\theta}}[\mathbf{g}_i])^2 \right]$$

High variance

- More **samples** needed
- Unstable training

## REINFORCE

- Simplest method to approximate policy gradient
- General and unbiased :)
- *Very high variance!* :(
- Not sample efficient



REINFORCE:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t G_t \nabla_{\theta} \log \pi_{\theta} (a_t | s_t) \right]$$

Reduce variance with **baseline**  $b_t$ :

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t (G_t - b_t) \nabla_{\theta} \log \pi_{\theta} (a_t | s_t) \right]$$

baseline

$$\begin{aligned} & \mathbb{E}_{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} [\mathbf{b}_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)] \\ &= \sum_{\mathbf{a}_t} \mathbf{b}_t \cancel{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} \frac{\nabla_{\theta} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\cancel{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}} \end{aligned}$$

$$\begin{aligned} & \mathbb{E}_{\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)}[\mathbf{b}_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)] \\ &= \sum_{\mathbf{a}_t} \mathbf{b}_t \cancel{\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)} \frac{\nabla_{\theta} \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)}{\cancel{\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)}} \\ &= \sum_{\mathbf{a}_t} \mathbf{b}_t \nabla_{\theta} \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t) = \mathbf{b}_t \nabla_{\theta} \sum_{\mathbf{a}_t} \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t) \\ &= \mathbf{b}_t \nabla_{\theta} 1 = 0 \end{aligned}$$

# WHAT BASELINE?

REINFORCE with **baseline**:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [\mathbf{R}_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t (\mathbf{G}_t - \mathbf{b}_t) \nabla_{\theta} \log \pi_{\theta} (\mathbf{a}_t | \mathbf{s}_t) \right]$$

Value function:

$$V^{\pi}(\mathbf{s}_t) = \mathbb{E}_{p(\tau|\pi, \mathbf{s}_t)} [\mathbf{G}_t]$$

Value function baseline:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [\mathbf{R}_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t (\mathbf{G}_t - V^{\pi}(\mathbf{s}_t)) \nabla_{\theta} \log \pi_{\theta} (\mathbf{a}_t | \mathbf{s}_t) \right]$$

# REINFORCE VS ACTOR-CRITIC

Act, receive reward.

How to reinforce?

$$V^{\pi}(s_t) = 0.63$$

REINFORCE:

I won!

Random reward



REINFORCE +  
baseline:

I won. That result is  
37% better than  
expected!

Increase of random  
reward wrt expected  
reward



# TRAINING VALUE FUNCTION

Like Deep Q-Learning, train neural network  $V_\phi$  with regression.

1. Use rollouts:

$$\phi \leftarrow \phi - \alpha \sum_{t=0}^{T-1} (V_\phi(s_t) - G_t)^2 \quad \text{target: reward-to-go}$$

2. Use bootstrapping (lower variance, biased):

$$\phi \leftarrow \phi - \alpha \sum_{t=0}^{T-1} (V_\phi(s_t) - \mathbb{E}(r_{t+1} + \gamma V_\phi(s_{t+1})))^2$$

target: bootstrapped  
expected reward-to-go

**REINFORCE with baseline:**

$$1. \tau \sim p(\tau|\theta)$$

$$2. \phi \leftarrow \phi - \alpha_c \sum_{t=0}^{T-1} (V_{\phi}(s_t) - \perp(r_{t+1} + \gamma V_{\phi}(s_t)))^2$$

$$3. \theta \leftarrow \theta + \alpha_a \sum_{t=0}^{T-1} \gamma^t (G_t - V_{\phi}(s_t)) \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$$

Discounted **reward** to-go

$$G = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'+1}$$

**Q-function** (**state-action** value function):

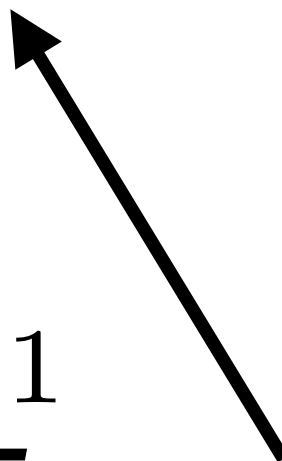
$$Q^{\pi}(s, a) = \mathbb{E}_{p(\tau|\pi, s, a)} [G]$$



# Q-FUNCTIONS IN POLICY GRADIENTS

$$Q^\pi(\mathbf{s}, \mathbf{a}) = \mathbb{E}_{p(\boldsymbol{\tau}|\pi, \mathbf{s}, \mathbf{a})} [G]$$

Policy gradient:

$$\begin{aligned} \nabla_{\boldsymbol{\theta}} \mathbb{E}_{p(\boldsymbol{\tau}|\boldsymbol{\theta})} [R_\gamma] &= \mathbb{E}_{p(\boldsymbol{\tau}|\boldsymbol{\theta})} \left[ \sum_{t=0}^{T-1} \gamma^t G_t \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t | \mathbf{s}_t) \right] \\ &= \mathbb{E}_{p(\boldsymbol{\tau}|\boldsymbol{\theta})} \left[ \sum_{t=0}^{T-1} \underbrace{\gamma^t Q^\pi(\mathbf{s}_t, \mathbf{a}_t)}_{\text{critic}} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t | \mathbf{s}_t) \right]_{\text{actor}} \end{aligned}$$


Much lower variance!

# NOTATION UPDATE

- Declutter notation:
- Current timestep  $t$ :  
 $r_t = r, a_t = a, s_t = s, G_t = G$
- Next timestep  $t + 1$ :  
 $r_{t+1} = r', a_{t+1} = a', s_{t+1} = s', G_{t+1} = G'$

Minimize **bootstrapped** error using regression:

$$\arg \min_{\theta} \left( \underbrace{Q_{\theta}(s_t, a_t)}_{\text{prediction}} - \underbrace{\mathbb{E}_{p(s_{t+1}, r_{t+1} | s_t, a_t)} [r_{t+1} + \gamma \max_{a_{t+1}} Q_{\theta}(s_{t+1}, a_{t+1})]}_{\text{target}} \right)^2$$

# REINFORCE VS ACTOR-CRITIC

Act, receive  
reward.

How to reinforce?

REINFORCE:

I won!

Random reward



Actor-critic:  
I think I'll win  
with 0.63  
probability!

Expected reward

Actor-critic:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t Q^{\pi}(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right]$$

Reduce variance even more with value function **baseline**:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t \left( \underbrace{Q^{\pi}(s_t, a_t)}_{\text{advantage}} - \underbrace{V^{\pi}(s_t)}_{\text{actor}} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right]$$

**Advantage** actor-critic



# ADVANTAGE VS Q-FUNCTION

Act, receive  
reward.

How to reinforce?

Actor-critic:  
I think I'll win  
with 63%  
probability!

Expected reward



Advantage  
actor-critic:  
I think I'll be 3%  
*more likely* to  
win.

Expected increase  
in reward

## Advantage actor-critic:

$$\nabla_{\theta} \mathbb{E}_{p(\tau|\theta)} [R_{\gamma}] = \mathbb{E}_{p(\tau|\theta)} \left[ \sum_{t=0}^{T-1} \gamma^t (Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right]$$

advantage  $A^{\pi}$

Estimate  $V^{\pi}$  with  $V_{\phi}$  (biased)

Estimate  $Q^{\pi}(s_t, a_t)$  with  $r_{t+1} + \gamma V_{\phi}(s_{t+1})$

$$A_{\phi}(s_t, r_{t+1}, s_{t+1}) = r_{t+1} + \gamma V_{\phi}(s_{t+1}) - V_{\phi}(s_t)$$

## **Advantage** actor-critic:

- Estimate advantage for current policy
- Use estimate to get improved policy

Like policy iteration, but with gentle steps



## Online Actor-Critic:

1.  $\mathbf{a} \sim \pi_{\theta}(\mathbf{a}|\mathbf{s})$

Select *actions* according to policy

2.  $\mathbf{s}', \mathbf{r}' \sim p(\mathbf{s}', \mathbf{r}'|\mathbf{s}, \mathbf{a})$

3.  $\Phi \leftarrow \Phi - \alpha_c (\mathbf{V}_{\Phi}(\mathbf{s}) - \perp(\mathbf{r}' + \gamma \mathbf{V}_{\Phi}(\mathbf{s}')))^2$

Update *critic*

4.  $\theta \leftarrow \theta + \alpha_a \mathbf{A}_{\Phi}(\mathbf{s}, \mathbf{r}', \mathbf{s}') \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}|\mathbf{s})$

Update *actor*

5.  $\mathbf{s} \leftarrow \mathbf{s}'$

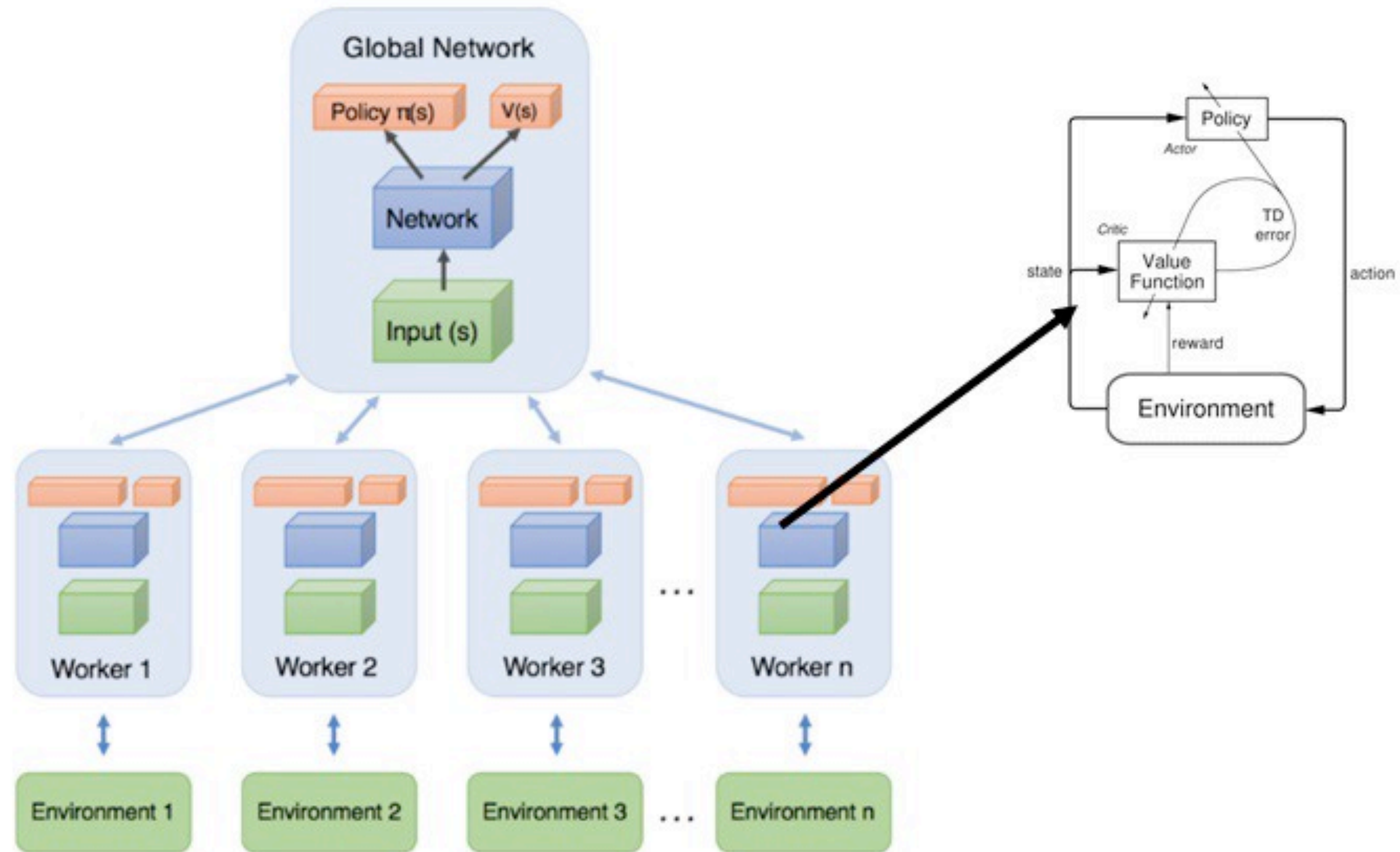
Uses only a single sample

And batching over time would give correlated minibatches

**A2C:** Multiple online agents

Collect experiences at each step for minibatch.

Efficient method!



# ADVANCED POLICY GRADIENT METHODS

- Take small steps in policy space
- Policy is *close* to another if KL-divergence is low
- Normal policy gradient: Difference in *parameter space*

$$\theta_{k+1} = \arg \max_{\theta} \bar{A}(\theta_k, \theta)$$

$$\text{s.t. } \bar{D}_{\text{KL}}(\theta \parallel \theta_k) \leq \epsilon$$

- How to ensure closeness?

$$\theta_{k+1} = \arg \max_{\theta} \bar{A}(\theta_k, \theta)$$

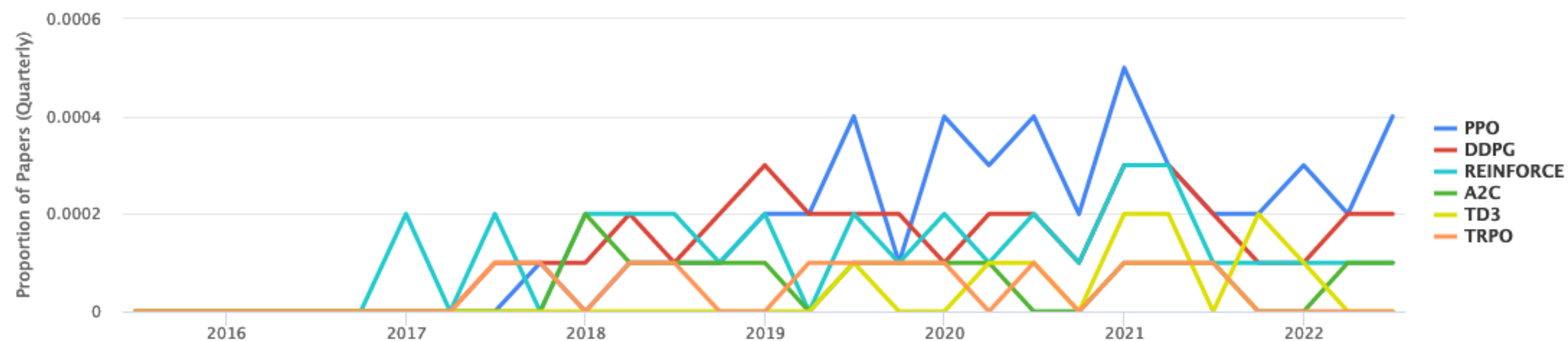
$$\text{s.t. } \bar{D}_{\text{KL}}(\theta \parallel \theta_k) \leq \epsilon$$

- How to ensure closeness?
- TRPO:
  - Uses *Natural Gradient*
  - Rescale AC-gradient by Fisher Information Matrix
  - Optimized using conjugate gradient
  - Complex to understand & implement well

$$\theta_{k+1} = \arg \max_{\theta} \bar{A}(\theta_k, \theta)$$

$$\text{s.t. } \bar{D}_{\text{KL}}(\theta \parallel \theta_k) \leq \epsilon$$

- How to ensure closeness?
- PPO Clip:
  - Clipped 'importance weights' between old and new policy
  - Discourages large policy changes
  - Simple to implement & popular!





Policy gradient methods:

Estimate gradient of *expected return*

Gradient estimation:

Estimate gradient of *any* expectation

$$\arg \max_{\theta} \mathbb{E}_{p_{\theta}(\mathbf{z})} [f(\mathbf{z})]$$

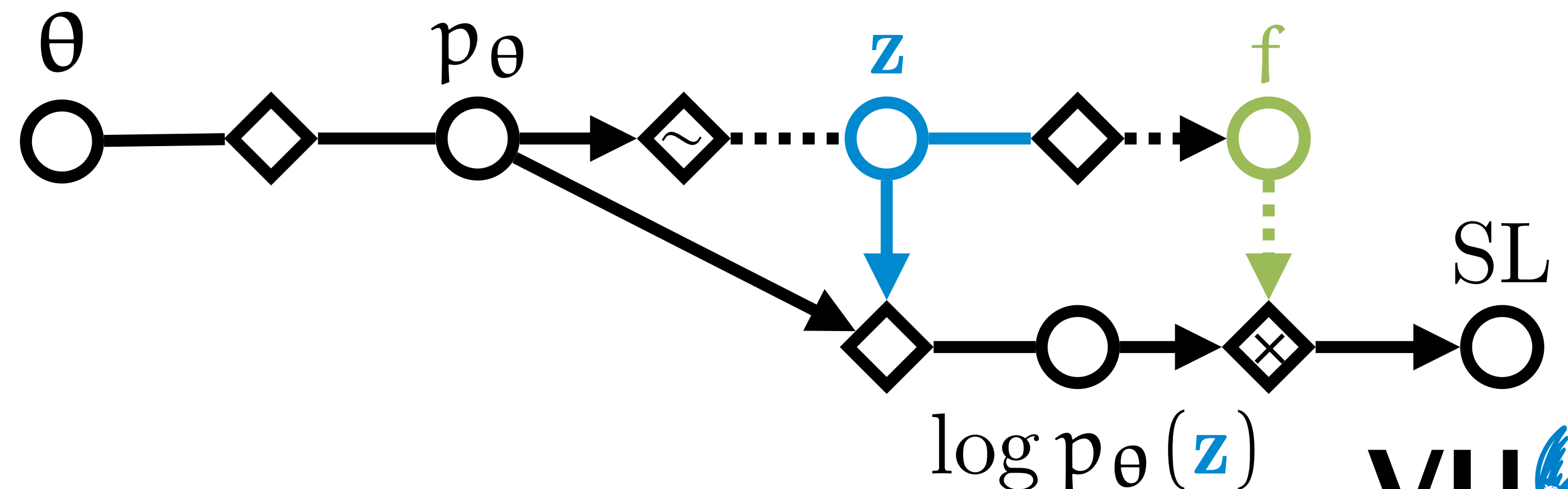


# SCORE FUNCTION

Recall score function:

$$\begin{aligned}\nabla_{\theta} \mathbb{E}_{p_{\theta}(\mathbf{z})} [f(\mathbf{z})] &= \mathbb{E}_{p_{\theta}(\mathbf{z})} \left[ f(\mathbf{z}) \frac{\nabla_{\theta} p_{\theta}(\mathbf{z})}{p_{\theta}(\mathbf{z})} \right] \\ &= \mathbb{E}_{p_{\theta}(\mathbf{z})} [f(\mathbf{z}) \nabla_{\theta} \log p_{\theta}(\mathbf{z})]\end{aligned}$$

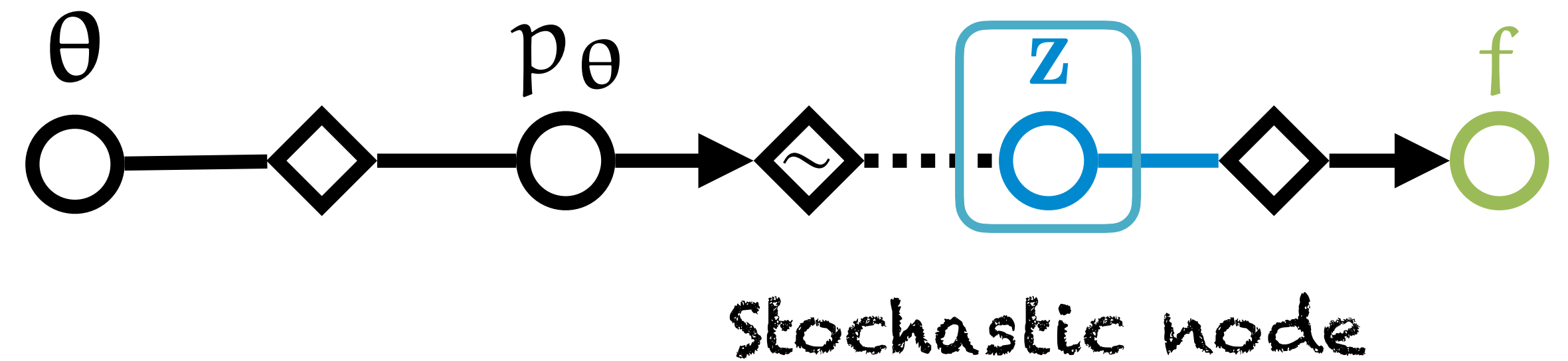
- All distributions  $p_{\theta}(\mathbf{z})$
- All functions  $f$
- But very high variance



# CAN WE DO BETTER?

Score function has high variance...

Can we do better?



## Reparameterization:

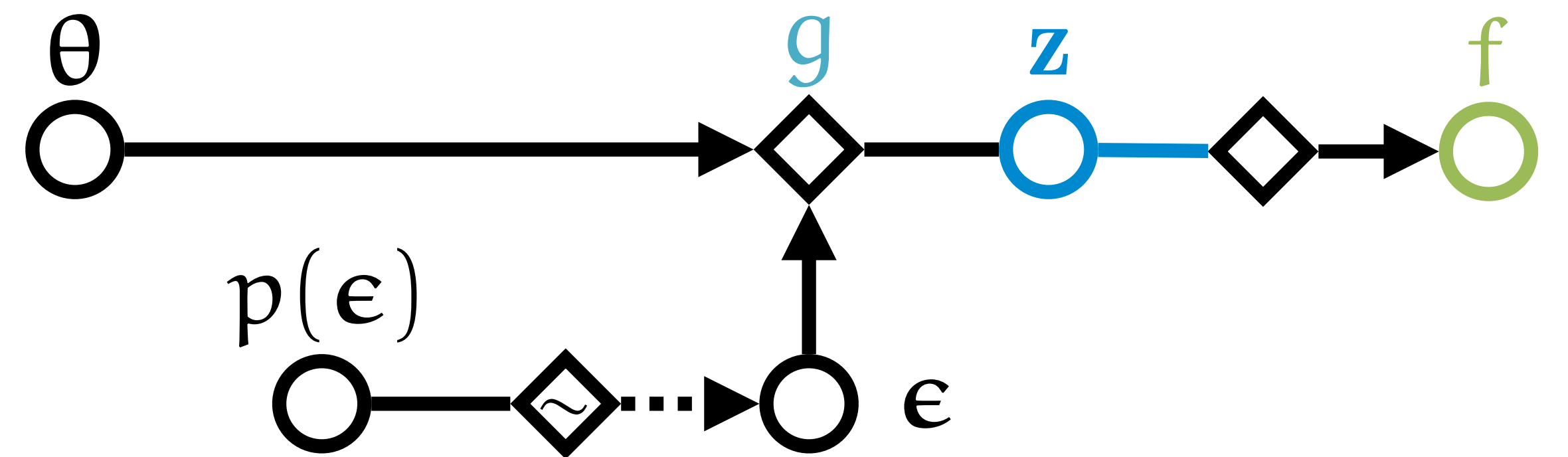
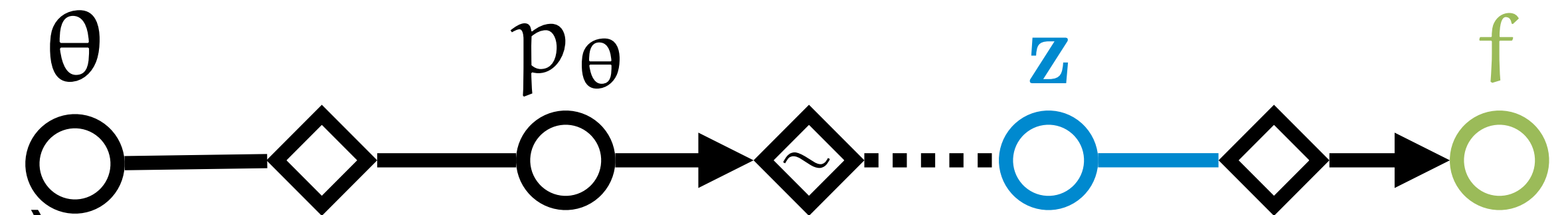
$$\mathbb{E}_{p_{\theta}(\mathbf{z})}[f(\mathbf{z})] = \mathbb{E}_{p(\epsilon)}[f(g(\theta, \epsilon))]$$

- Noise distribution  $p(\epsilon)$

$$\mathbf{z} = g(\theta, \epsilon) \sim p_{\theta}(\mathbf{z})$$

Pathwise derivative (=backprop):

$$\mathbf{g}_{PD} = \frac{\partial f}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \theta}, \quad \epsilon \sim p(\epsilon)$$



# PATHWISE DERIVATIVE

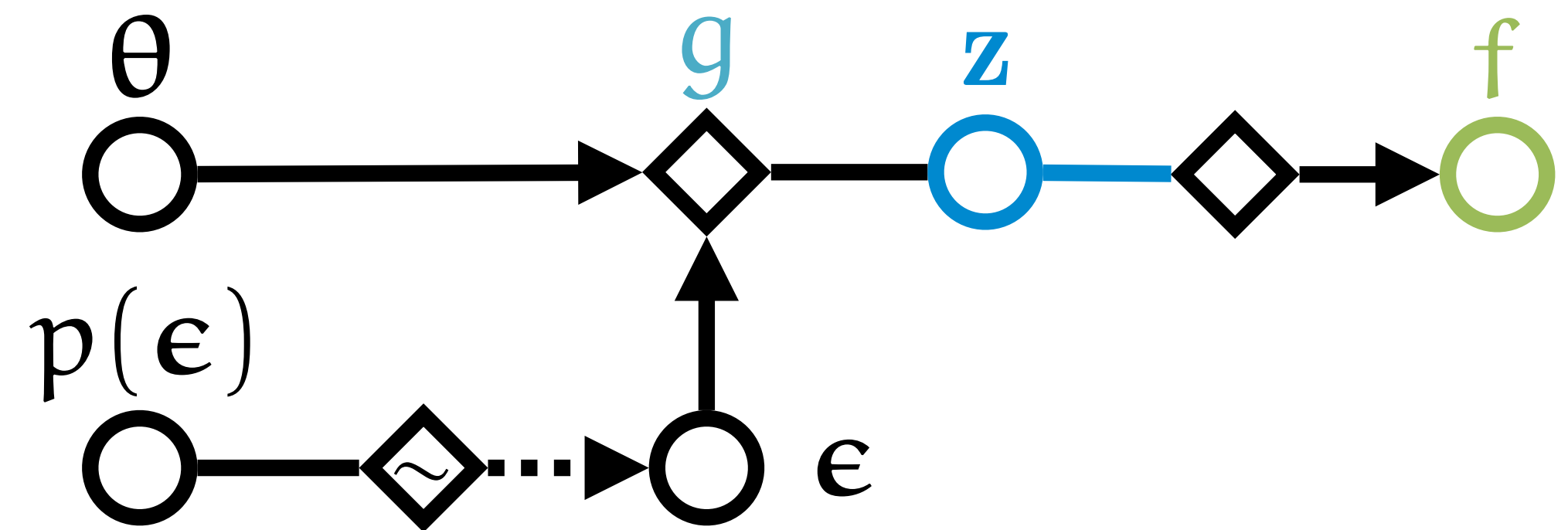
Low variance :)

- Uses extra info:  $\frac{\partial f}{\partial \mathbf{z}}$

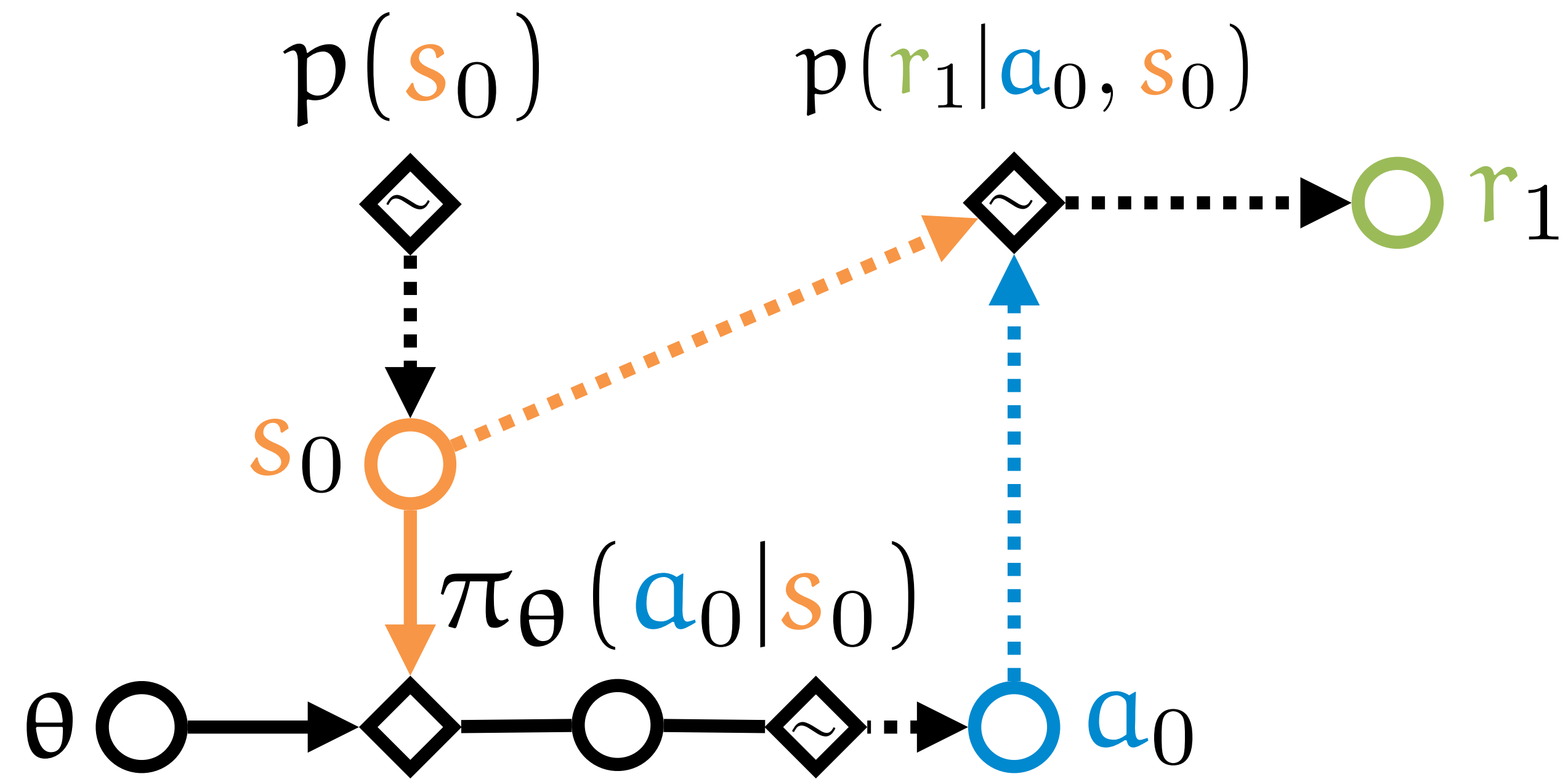
Requires:

- Differentiable function  $f$  :(
- Appropriate *continuous* distribution  $p_{\theta}(\mathbf{z})$  :(

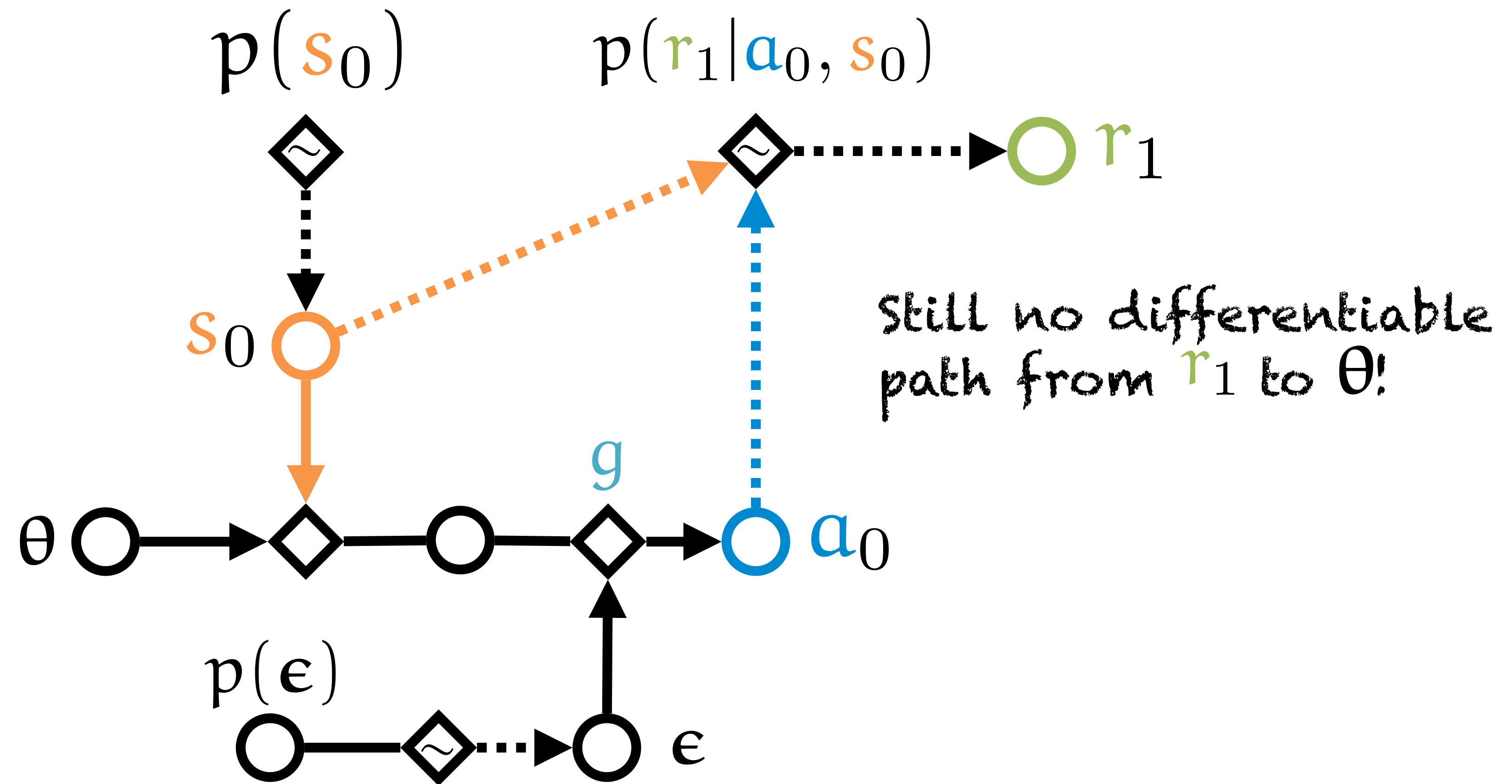
No reparameterization for *discrete* distributions



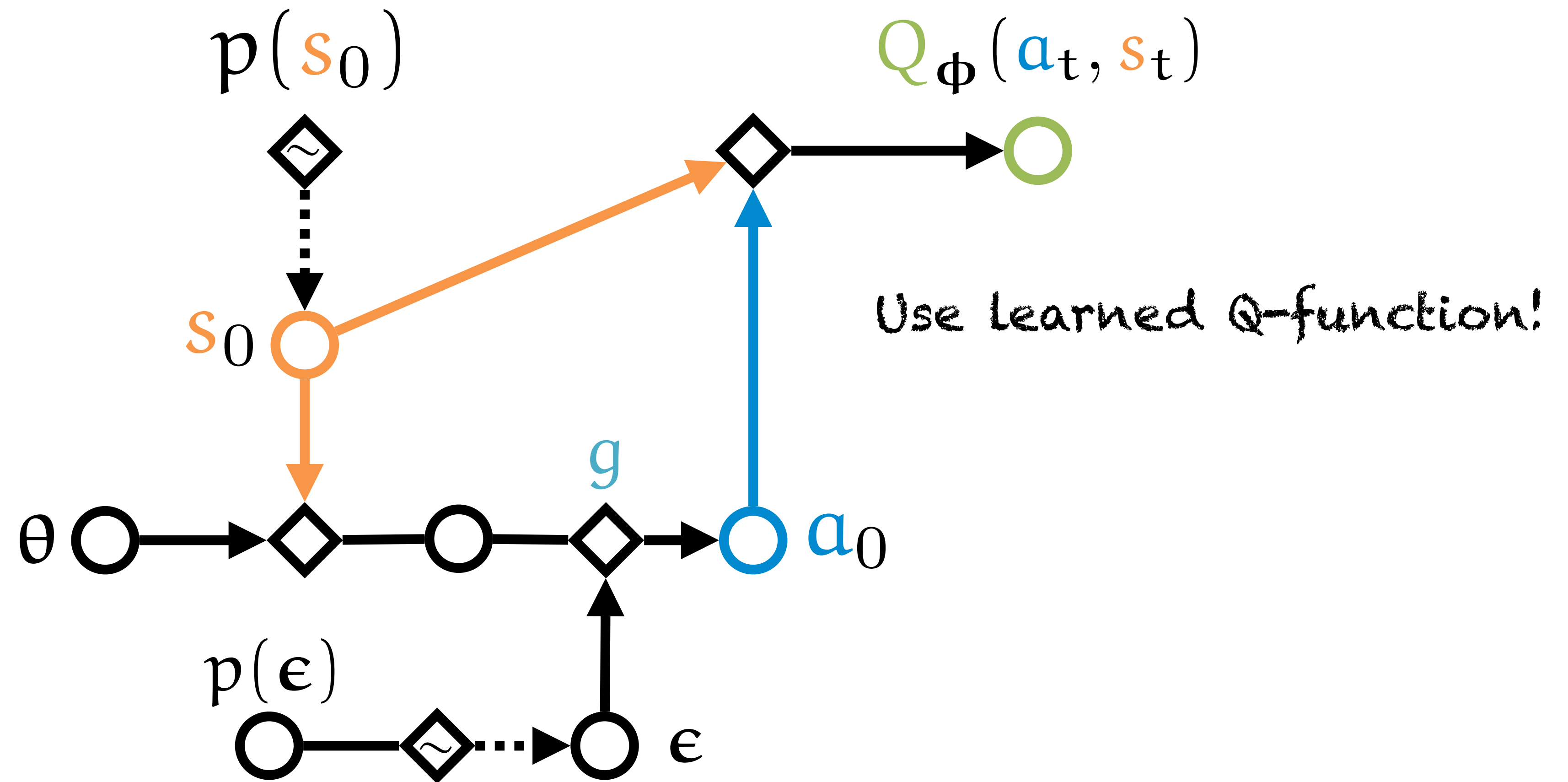
# REPARAMETERIZATION IN RL?



# REPARAMETERIZATION IN RL?



# REPARAMETERIZATION IN RL!



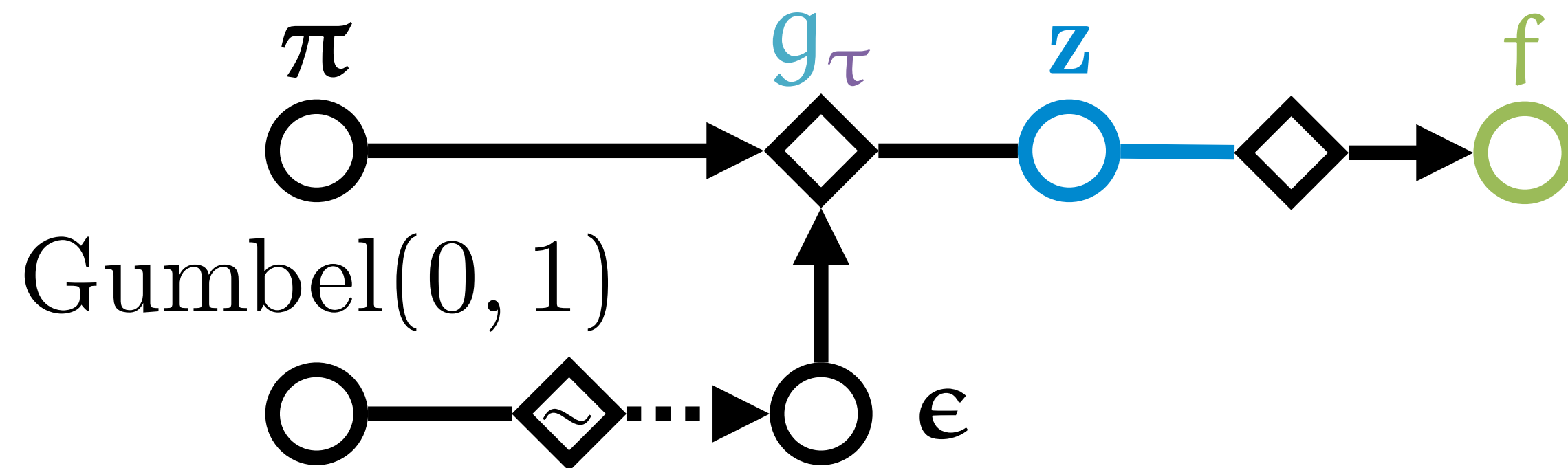


# GUMBEL SOFTMAX

Probabilities  $\pi_1, \dots, \pi_K$ , temperature  $\tau > 0$

$\epsilon_1, \dots, \epsilon_K \sim \text{Gumbel}(0, 1)$

$\mathbf{z} = \mathbf{g}_\tau(\boldsymbol{\epsilon}, \boldsymbol{\pi}) = \text{Softmax}((\log \boldsymbol{\pi} + \boldsymbol{\epsilon})/\tau)$



**Storchastic:** Define computation graph with sampling steps.

Compute gradient estimators *automatically!*

- PyTorch library with easy API
- Many low-variance estimators implemented
- Focus on discrete distributions

- Off-policy algorithm
- Uses reparameterization to maximize through critic
- Adds **entropy-regularization**
  - Encourage exploration
- Similar algorithms
  - Deep Deterministic Policy Gradient (DDPG)
  - Twin Delayed DDPG (TD3)

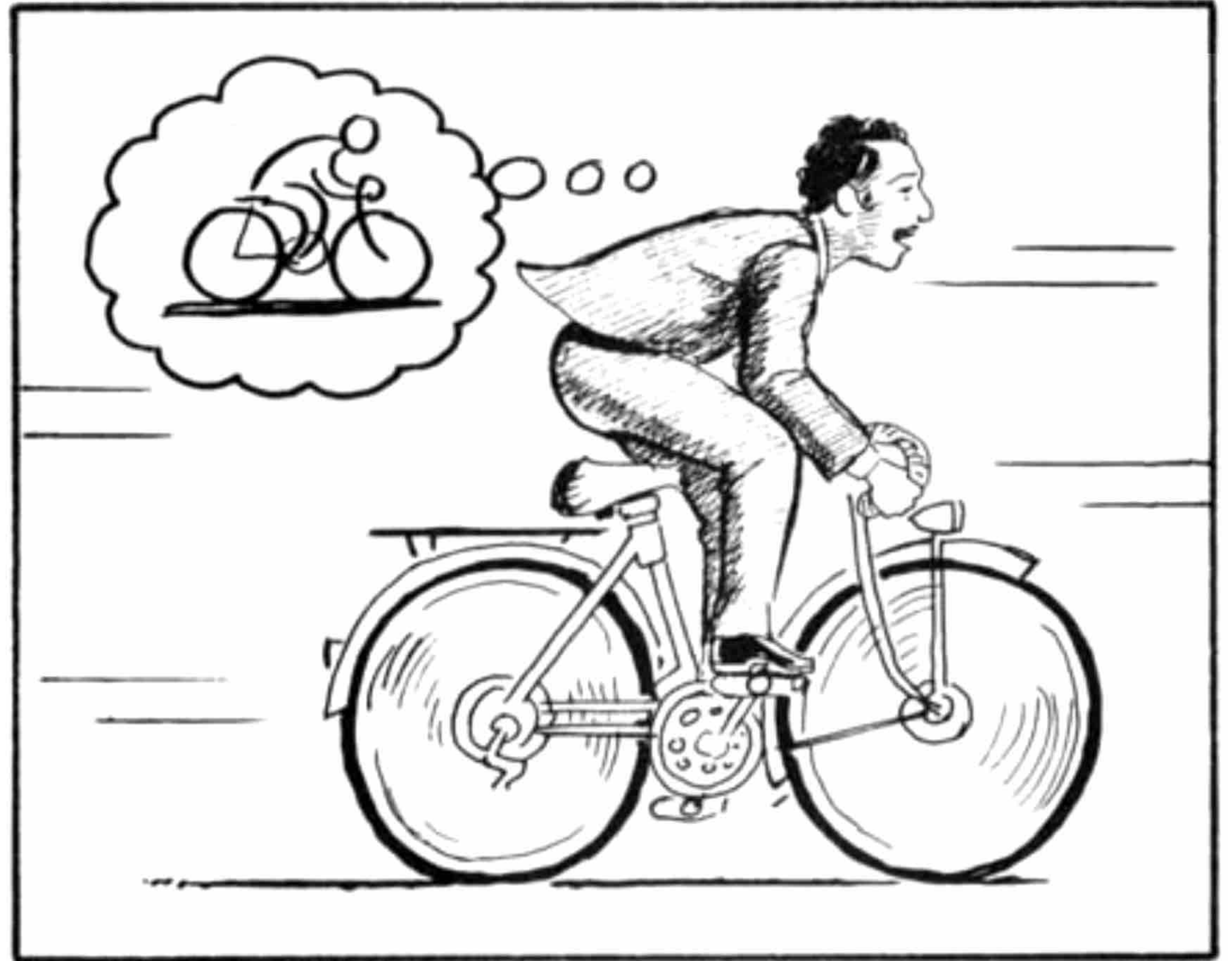
## Actor-critic methods

- Model **actor**: Policy NN
- Model **rewards**: **Value** function NN

What about 3rd RL component: **environment**?

## World Models

- Model **environment** using neural networks!

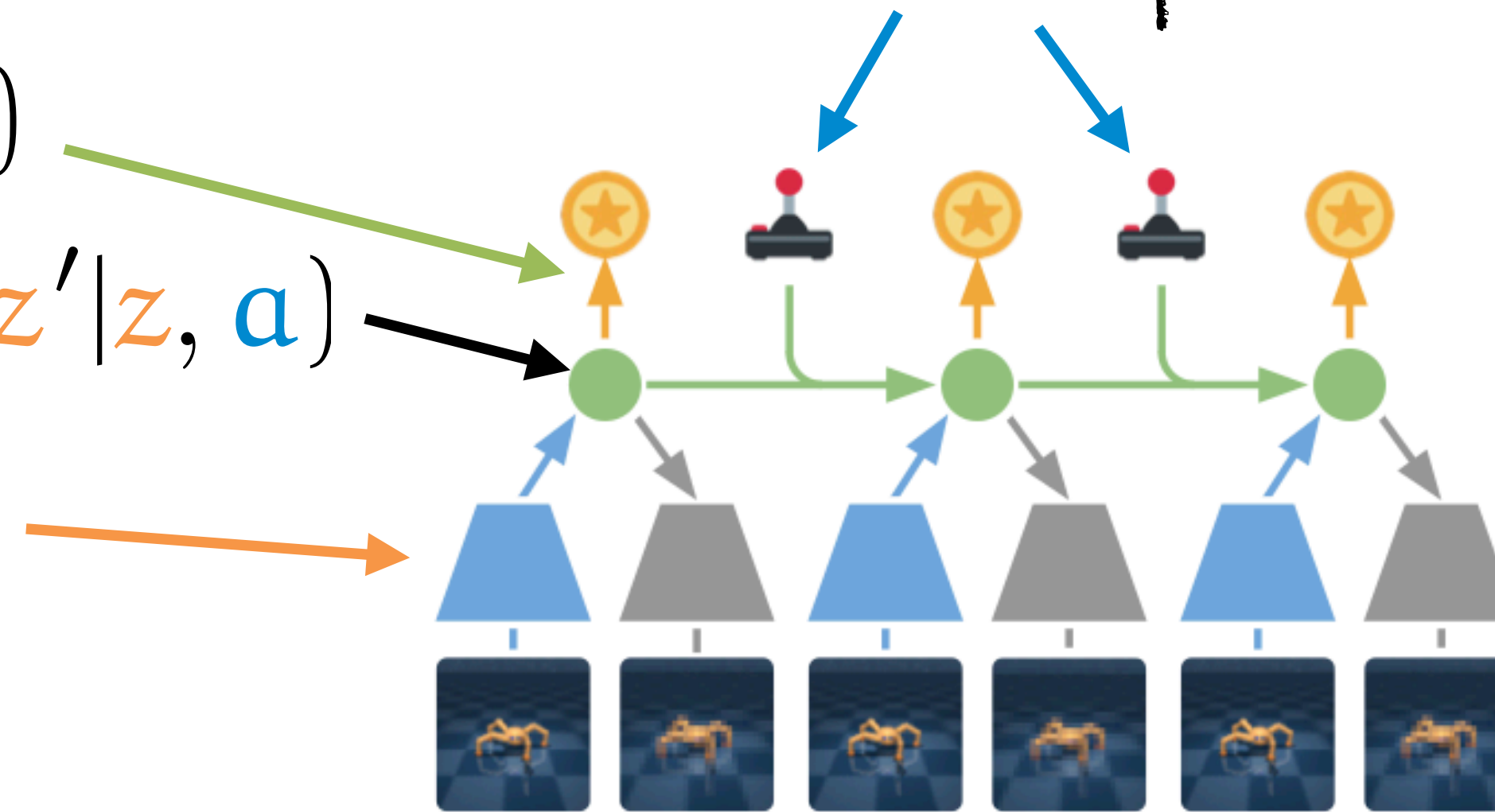


McCloud, Scott. *Understanding Comics: The Invisible Art*. Tundra Publishing, 1993.  
Ha, David, and Jürgen Schmidhuber. "World models."

# WORLD MODEL COMPONENTS

Components of our World Model: **Action inputs**

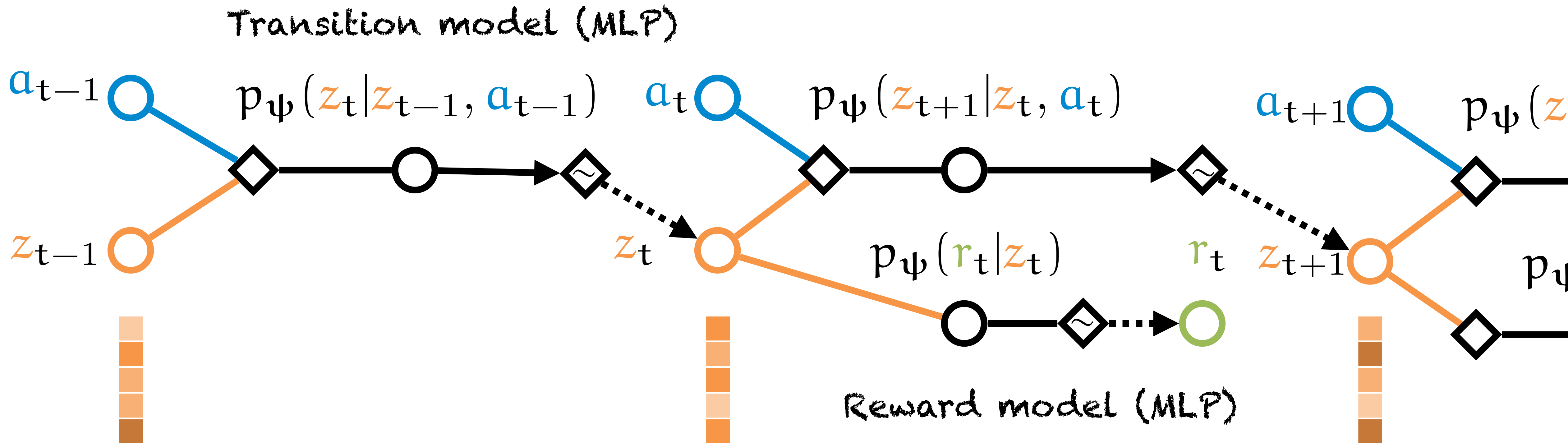
- Reward model  $p_{\psi}(r|z)$
- Transition model  $p_{\psi}(z'|z, a)$
- **State encoder**  $q_{\psi}(z|s)$



Hafner, Danijar, et al. "Dream to Control: Learning Behaviors by Latent Imagination." International Conference on Learning Representations. 2019.



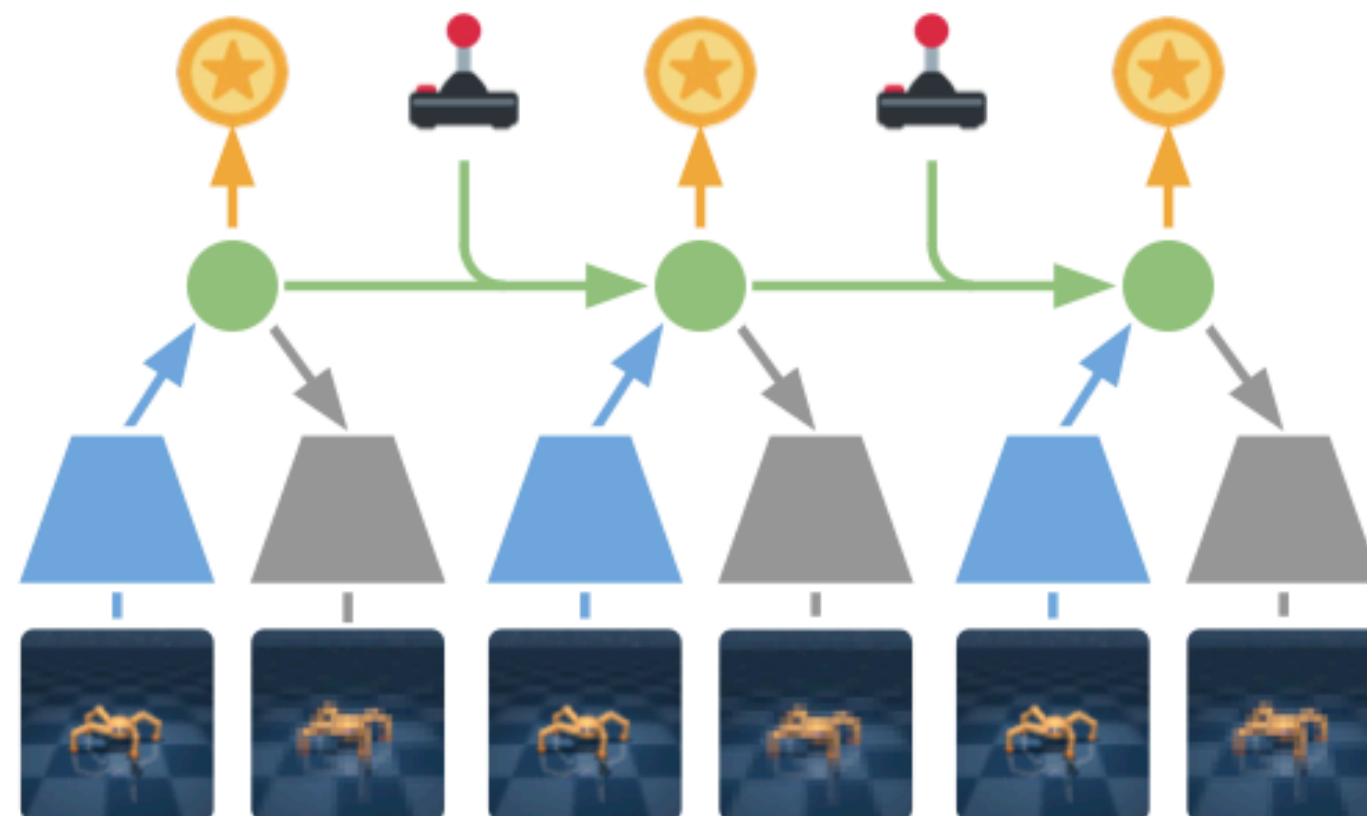
# TRANSITION DYNAMICS





World models can be used to **dream** trajectories  
(more formally, **latent imagination**)

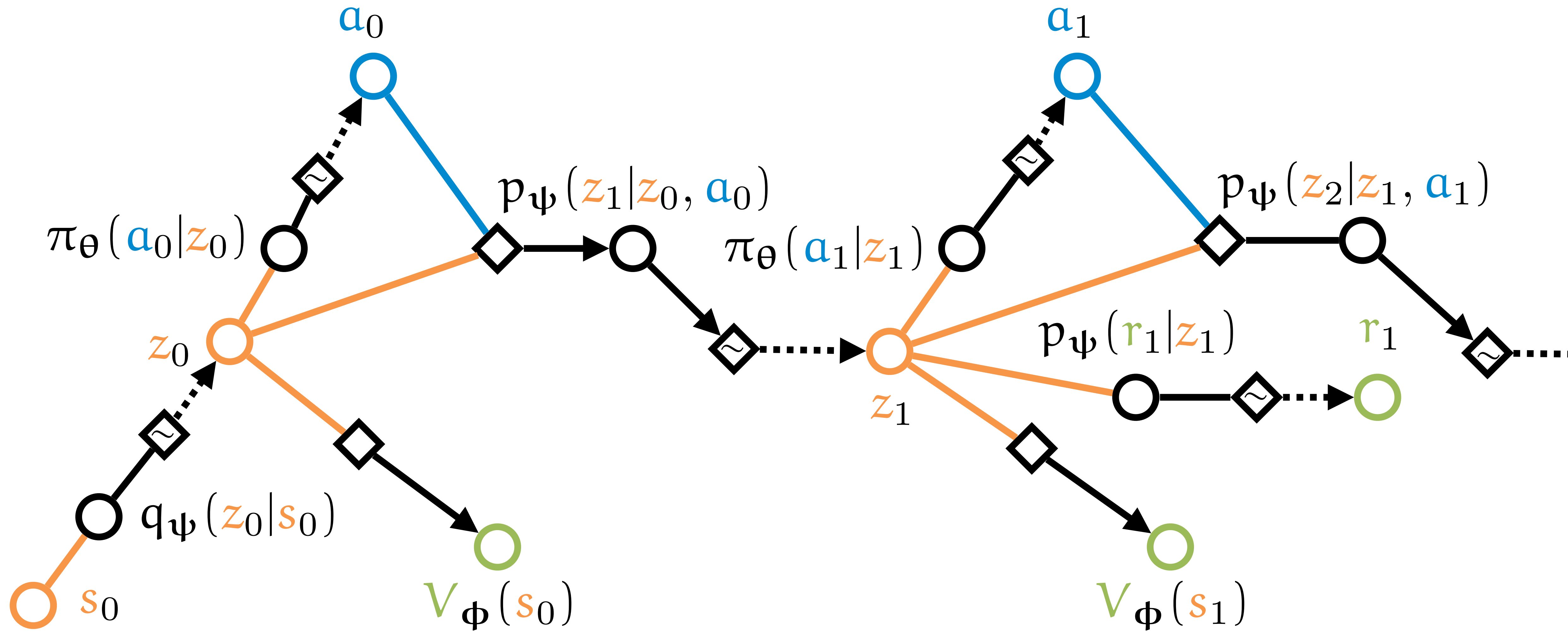
Use to train RL agents  $\pi_{\theta}(a|z)$  *without interaction with environment*



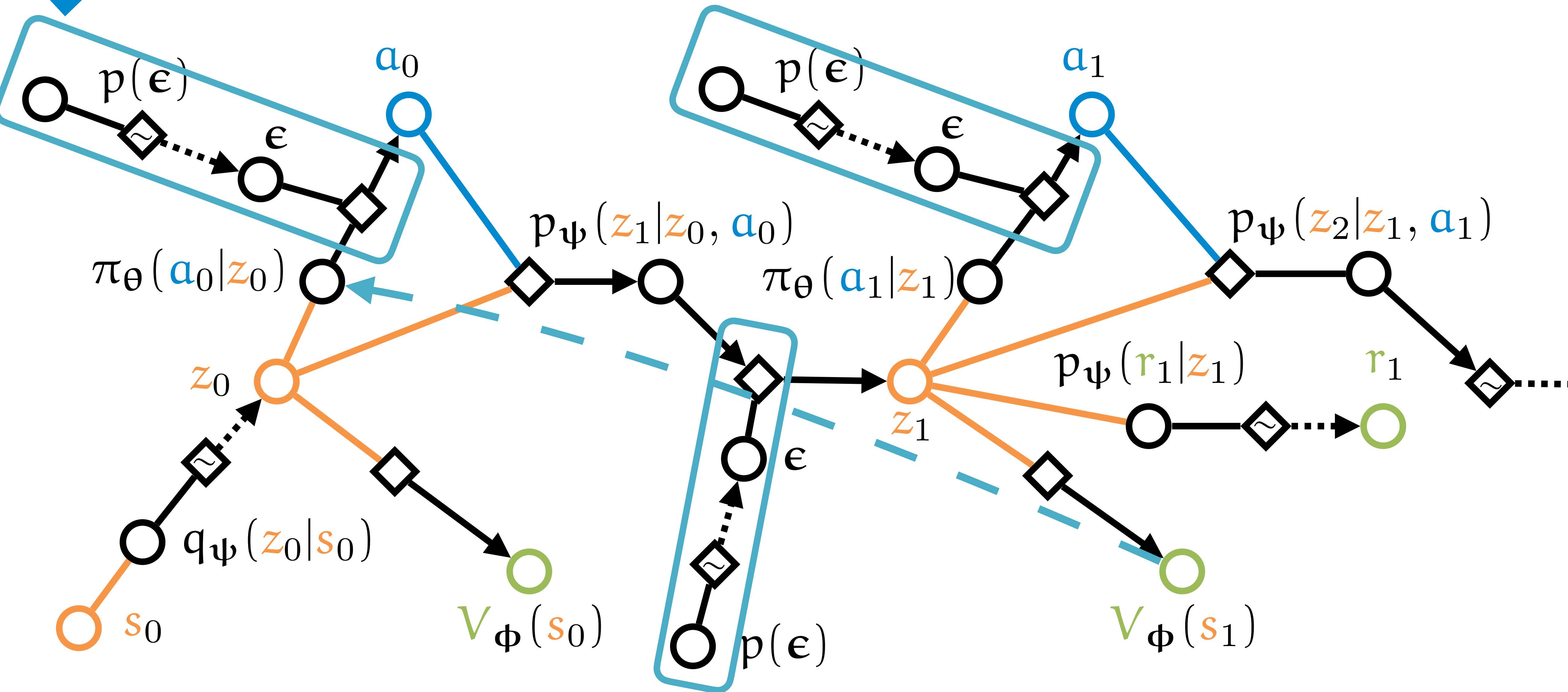
Hafner, Danijar, et al. "Dream to Control: Learning Behaviors by Latent Imagination."  
International Conference on Learning Representations. 2019.



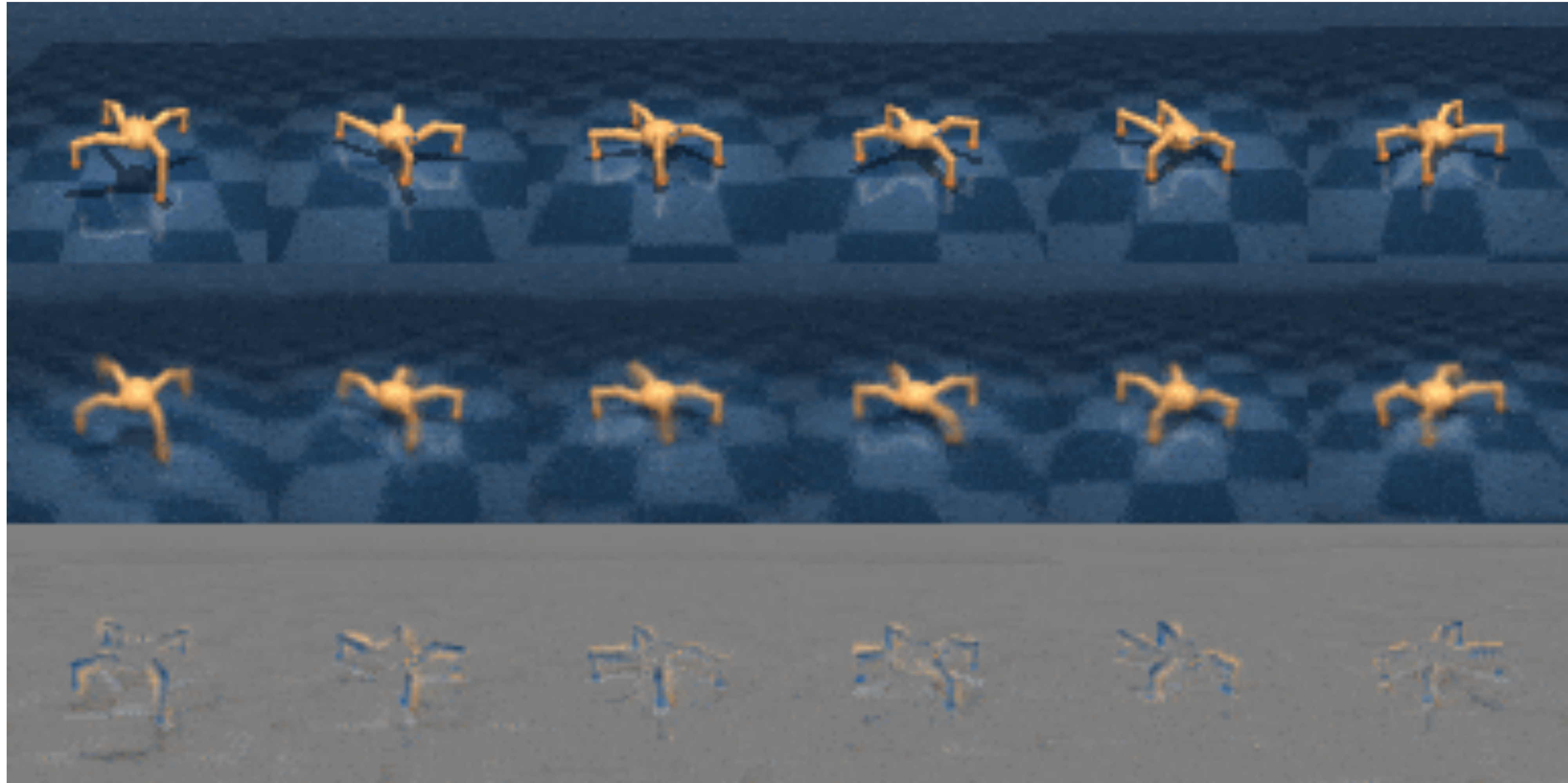
# TRAINING BY DREAMING



# TRAINING BY BACKPROPAGATING THROUGH DREAM







Hafner, Danijar, et al. "Dream to Control: Learning Behaviors by Latent Imagination." International Conference on Learning Representations. 2019.

# SUMMARY

- **REINFORCE**: Basic policy gradient algorithm
- **Actor-critic**: Add critic to reduce variance
- **TRPO/PPO**: Ensure small and controlled learning steps
- **SAC**: Use reparameterization and entropy regularization
- **World Models**: Train policy inside learned model

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# THANK YOU!

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<https://github.com/HEmile/storchastic>